Design and Construction of a Helmholtz Coil

Project Members:

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ABSTRACT

The purpose of our project was to construct a Helmholtz Coil that generates a uniform magnetic field strength less than 5% of the Earth's magnetic field. In addition, two non-contact sensors were also designed in order to analyze the magnitude and direction of the magnetic field within the Helmholtz Coil. The design of both the Helmholtz Coil and sensors were analyzed and implemented using household materials.

INTRODUCTION

The Helmholtz Coil is a classic configuration of using two identical coils of wire to induce a magnetic field. The design was chiefly named after its curator the German physicist, Hermann von Helmholtz. The two coils share the same radius and sit symmetrically on the same axis. The distance of separation between the coils is that of the radius of each coil. A current shall flow through each coil (both in the same direction) and will create a uniform magnetic field radially through the space existing between the two coils. The desired effect is a varying magnetic field which exists between the coils along the axis of symmetry they share (i.e., the zaxis).

CONSTRUCTION OF INDIVIDUAL COMPONENTS

The apparatus for the Helmholtz Coil is composed of medium-density fiberboard (MDF) which makes for an excellent insulating material. It consists of a base, two supports, and two cylinders which are located on the wall supports. The base dimensions are 36.5 cm L x 33.5 cm W. The wall dimensions are 30 cm H x 28 cm W. The cylinders each have a radius of 12 cm. The wall supports are separated by 16 cm with a distance of 12 cm from the outside of one coil to the inside of the other coil. Since the radius of each coil is equal to their distance of separation,

this should help create a uniform magnetic field. The parameters chosen for the apparatus were based upon having enough space to include sensors. Please refer to Figures 1, 2, and 3 of Appendix A.

The Earth's magnetic field, B_{Earth} , was found by using the longitude, latitude, and elevation of Las Vegas using reference #1 to be 48.445µT. The Helmholtz Coil was built using a 5.6V battery (5.6V instead of 6V since we used it so much for testing purposes), 1.5k Ω resistor (measured using a multimeter), and N = 88 turns per coil. The magnetic field of the system was calculated using Equation 1 of Appendix B. The coil system was modeled in MATLAB in order to determine what the magnetic field within our 2cm cylinder would be.

Sensor #1: Solenoid

The first sensor is a homemade solenoid, with a needle placed on the inside to detect the strength of the field inside the solenoid. Due to current moving through a wire wrapped around a tube, the current produces a magnetic field inside the solenoid. The field is mostly uniform due to the geometry of the solenoid since its length is much larger than its radius. Please refer to Appendix A Figure 5 and Appendix B Equation 3.

$$\boldsymbol{B}_{\boldsymbol{s}} = \frac{\mu_0 N_s I_s}{L_s}$$

Where Bs is the magnetic field in the solenoid, N_s is number of turns around tube, I_s is current,

and *Ls* is length of the solenoid.

$$B_s = 108\mu T$$
, $\mu_0 = 4\pi * 10^{-7} H/m$, $L_s = 10 \text{cm} = 0.01 \text{m}$, $N_s = 119 \text{ turns}$
 $I_s = \frac{V}{R_s} = \frac{5.6V}{78\Omega} = 72.73 \text{mA}$

Using vector addition, we can determine the magnetic field strength of the Helmholtz Coil, based on the angle of deviation of the needle. The magnitude of the magnetic field strength can also be determined through geometry by measuring the angle of deviation between the magnetic field of the solenoid and the Helmholtz coil. We also observe the relationship between the Magnetic Field **B** and the current **I**. **B** and **I** are proportional.

Sensor #2: Compass

A compass was designed in order to detect the magnetic field in the region between the coils. The compass was built using a glass container, a cork, and a needle that had been magnetized. The magnetized needle was pushed through the center of the cork and placed in the container of water. A small divot was drilled into the container in order to keep the cork and needle centered in the glass. The cork was chosen to hold the needle because it floats on water (less dense than water). Once the needle has been magnetized it acts like a small magnet. Since it is free to rotate with minimal resistance in the container, the needle can detect the earth's magnetic field and align itself to point north. Inside the Helmholtz coil the compass points to magnetic north when there is no current applied through the wiring of the coil. When the coil is connected to the 5.6V battery the needle points in the direction of the external magnetic field.



When θ is 0° the coil has no current flowing through it and the compass points in the direction of earth's magnetic field. As the current through the Helmholtz coil increases so does θ . The net change in the magnetic field can be found by using vector addition or by measuring the angle of deviation. The compass points in the direction of the resultant magnetic field between the earth's field and the magnetic field generated by applying a current through the Helmholtz coil. Please refer to Appendix A Figure 4.

Resistor Design:

To create a resistor, you must start with the equation for resistance $R = \frac{\rho L}{A}$ where A is the cross-sectional area, L is the length of resistor, and ρ is the resistivity of the material. We started by looking at the resistivity of many different wires and found that graphite and kanthal A-1 wire have high resistivities.

Graphite makes a great resistor but is very unpredictable. The resistance is about $1k\Omega$ per inch when measured along a straight line. Then we tried diverse types of paper and different grades of graphite. We noticed that the rougher the paper the less resistance per square inch. The resistance was also less per square inch with the softer grades of graphite.

Kanthal wire is a material with a lower resistivity than graphite but still relatively high . This wire is about $1k\Omega$ per 1 inch of 32-gauge wire. This material makes a stable resistance value and changes very little. And rew used about 16 feet of this wire and tightly wrapped the wire around a metal rod. The wire was then loosened and was slid off the rod to make a compact slinky made of kanthal wire to make a total resistance of 266Ω . We then cut this up into several resistors for the solenoid and the Helmholtz coil. Please refer to Appendix A Figure 6.

TESTING AND RESULTS

We first tested our magnetic field with our compass. Prior to placing the compass in between the two coils, the needle in our compass, floating in water, was magnetized in order to point towards magnetic north. The Helmholtz Coil was oriented to where the magnetic field vector between the coils was perpendicular to that of the Earth's magnetic field. When placed inside the Helmholtz Coil, the orientation of the needle is rotated due to a torque by the force produced by our two coils. This verified proper performance of the coil.

Using our equations, we constructed the Solenoid to create a magnetic field that is twice the amount of the field produced by our Helmholtz Coil. In theory and based on vector addition, if the field in the solenoid is twice the amount of the Helmholtz Coil, with fields aligned perpendicularly, the needle inside our solenoid rotates to a 22.5° angle. Observing our needle in demonstration, we are not at this angle although we do assume this is due to torsion in the thread holding the needle.

CONCLUSION

Using only household materials, we were able to apply our knowledge of magnetostatics to create a functioning Helmholtz coil, a homemade resistor, and two sensors detecting our field inside our coil. Based on calculations we designed a Helmholtz coil to function within less than 5% of the earth's magnetic field. We used our theory and equations to create a working Helmholtz coil (above the Earth's Magnetic Field), and with our sensors we were able to measure the strength of our field. The greatest challenges of our project was taking our equations for the coil and field and implementing those results into our design while remaining within the constraints required for the design.

REFERENCES

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Appendix A: Photos



Figure 1: Helmholtz Coil Version 1



Figure 2: Helmholtz Coil Version 2



Figure 3: AutoCAD rendering of apparatus



Figure 4: Sensor 1 - Compass



Figure 5: Sensor 2 - Solenoid



Figure 6: Resistors



Figure 7: Jacob Reed with the design of apparatus



Figure 8: Cody McDonald with the design of apparatus



Figure 9: Moriah Wingrove with design of compass



Figure 10: Elizabeth Heider with design of solenoid



Figure 11: Andrew Buchanan with design of resistors



Figure 12: From left to right - Cody, Jake, Moriah, Liz, Andrew

Appendix B: Lengthy calculations and MATLAB Simulations

Helmholtz Coils Calculations – Coil 1 and Coil 2

Coil ₁ has N₁ turns of current filament, radius a_1 , and is located in the $z = d_1$ plane Coil ₂ has N₂ turns of current filament, radius a_2 , and is located in the $z = d_2$ plane

Solving for the magnetic flux density in the top coil at a point p off the z-axis:

$$\vec{B}(\vec{r}_p) = \vec{V}_p \times A_1(\vec{r}_p)$$
$$d\vec{A}_1 = \frac{\mu_0}{4\pi} \left[\frac{I_{c_1} d\vec{l}_{s_1}}{R} \right]$$
$$\vec{B}(\vec{r}_p) = \vec{V}_p \times \frac{\mu_0}{4\pi} \oint \frac{I_{c_1} d\vec{l}_{s_1}}{R}$$

Field in the z-axis does not equal zero

Where the current in Coil 1 depends on the current and number of filaments

$$I_{c_1} = I_1 N_1$$

Solving for $d\vec{l}_{s_1}$:

$$d\vec{l}_{s_1} = dr_{s_1}\hat{r}(\varphi_s) + r_{s_1}d\varphi_s\hat{\varphi}(\varphi_s) + dz_{s_1}\hat{z}$$
$$dr_{s_1}\hat{r}(\varphi_s) = 0$$
$$dz_{s_1}\hat{z} = 0$$
$$d\vec{l}_{s_1} = r_{s_1}d\varphi_s\hat{\varphi}(\varphi_s)$$
$$r_{s_1} = a_1$$
$$d\vec{l}_{s_1} = a_1d\varphi_s\hat{\varphi}(\varphi_s)$$

Solving for R:

$$R = |\vec{R} \cdot \vec{R}|^{\frac{1}{2}}$$

$$\vec{R} = \vec{r}_{p} - \vec{r}_{s}$$

$$\vec{r}_{p} = r_{p}\hat{r}(\varphi_{p}) + z_{p}\hat{z}$$

$$\vec{r}_{s} = r_{s}\hat{r}(\varphi_{s}) + z_{s}\hat{z}$$
Using
$$r_{s} = a_{1} \text{ and } z_{s} = d_{1}$$

$$\vec{r}_{s} = a_{1}\hat{r}(\varphi_{s}) + d_{1}\hat{z}$$

$$\vec{R} = r_{p}\hat{r}(\varphi_{p}) + z_{p}\hat{z} - (a_{1}\hat{r}(\varphi_{s}) + d_{1}\hat{z})$$

$$\vec{R} = r_{p}\hat{r}(\varphi_{p}) + z_{p}\hat{z} - a_{1}\hat{r}(\varphi_{s}) - d_{1}\hat{z}$$

$$\vec{R} = r_{p}\hat{r}(\varphi_{p}) - a_{1}\hat{r}(\varphi_{s}) + (z_{p} - d_{1})\hat{z}$$

$$R = |r_{p}\hat{r}(\varphi_{p}) - a_{1}\hat{r}(\varphi_{s}) + (z_{p} - d_{1})\hat{z} \cdot [r_{p}\hat{r}(\varphi_{p}) - a_{1}\hat{r}(\varphi_{s}) + (z_{p} - d_{1})\hat{z}]|^{\frac{1}{2}}$$

$$R = \left(\left[r_p \hat{r}(\varphi_p) \cdot r_p \hat{r}(\varphi_p) \right] + \left[r_p \hat{r}(\varphi_p) \cdot -a_1 \hat{r}(\varphi_s) \right] + \left[r_p \hat{r}(\varphi_p) \cdot (z_p - d_1) \hat{z} \right] \\ + \left[-a_1 \hat{r}(\varphi_s) \cdot r_p \hat{r}(\varphi_p) \right] + \left[-a_1 \hat{r}(\varphi_s) \cdot -a_1 \hat{r}(\varphi_s) \right] + \left[-a_1 \hat{r}(\varphi_s) \cdot (z_p - d_1) \hat{z} \right] \\ + \left[(z_p - d_1) \hat{z} \cdot r_p \hat{r}(\varphi_p) \right] + \left[(z_p - d_1) \hat{z} \cdot -a_1 \hat{r}(\varphi_s) \right] \\ + \left[(z_p - d_1) \hat{z} \cdot (z_p - d_1) \hat{z} \right]^{\frac{1}{2}}$$

$$R = \left[r_p^2 - 2a_1 r_p \left(\hat{r}(\varphi_p) \cdot \hat{r}(\varphi_s) \right) + a_1^2 + \left(z_p - d_1 \right)^2 \right]$$
$$\hat{r}(\varphi_p) \cdot \hat{r}(\varphi_s) = \left| \hat{r}(\varphi_p) \right| \left| \hat{r}(\varphi_s) \right| \cos(a)$$

Where a is the angle between $\hat{r}(\phi_p)$ and $\hat{r}(\phi_s)$



$$\hat{r}(\varphi_p) \cdot \hat{r}(\varphi_s) = cos(\varphi_p - \varphi_s)$$

Plugging this back into R gives

$$R = \left[r_p^2 + a_1^2 - 2a_1r_p\cos(\varphi_p - \varphi_s) + (z_p - d_1)^2\right]^{\frac{1}{2}}$$

The calculated values of $R, d\vec{l}_{s_1}, I_{c_1}$ were plugged back into the magnetic flux density equation

$$\vec{B}(\vec{r}_p) = \vec{\nabla}_p \times \frac{\mu_0}{4\pi} \oint \frac{I_1 N_1 a_1 d\varphi_s \hat{\varphi}(\hat{\varphi}_s)}{\left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2\right]^{\frac{1}{2}}}$$

$$\vec{B}(\vec{r}_p) = \frac{\mu_0 I_1 N_1 a_1}{4\pi} \int_{\varphi_s=0}^{2\pi} \vec{V}_p \times \frac{d\varphi_s \hat{\varphi}(\hat{\varphi}_s)}{\left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2\right]^{\frac{1}{2}}}$$

In order to perform the cross product $\widehat{\varphi}(\widehat{\varphi}_s)$ must be a function of p:

$$\hat{\varphi}(\varphi_s) = A\hat{r}(\varphi_p) + B\hat{\varphi}(\varphi_p) + C\hat{z}$$



Cos(G-F) = cos(G)cos(F) + sin(G)sin(F)

$$\cos\left(\frac{\pi}{2} - (\varphi_p - \varphi_s)\right) = \cos\left(\frac{\pi}{2}\right)\cos(\varphi_p - \varphi_s) + \sin\left(\frac{\pi}{2}\right)\sin(\varphi_p - \varphi_s)$$
$$\cos\left(\frac{\pi}{2}\right) = 0$$
$$\sin\left(\frac{\pi}{2}\right) = 1$$
$$\cos\left(\frac{\pi}{2} - (\varphi_p - \varphi_s)\right) = \sin(\varphi_p - \varphi_s)$$
$$A = \sin(\varphi_p - \varphi_s)$$

$$B = \hat{\varphi}(\varphi_s) \cdot \hat{\varphi}(\varphi_p)$$
$$B = \cos(\varphi_p - \varphi_s)$$

$$C = \hat{\varphi}(\varphi_s) \cdot \hat{z}$$
$$C = 0$$
$$\hat{\varphi}(\varphi_s) = \sin(\varphi_p - \varphi_s) \hat{r}(\varphi_p) + \cos(\varphi_p - \varphi_s) \hat{\varphi}(\varphi_p)$$

Plugging $\widehat{\phi}(\phi_s)$ as a function of ϕ_p back into magnetic flux density equation

$$\vec{B}(\vec{r}_p) = \frac{\mu_0 I_1 N_1 a_1}{4\pi} \int_{\varphi_s=0}^{2\pi} \vec{V}_p \times \frac{\sin(\varphi_p - \varphi_s) \hat{r}(\varphi_p) + \cos(\varphi_p - \varphi_s) \hat{\varphi}(\varphi_p)}{\left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2\right]^{\frac{1}{2}}} d\varphi_s$$

Solving for the cross product:

The cross product for cylindrical coordinates is given by the following equation:

$$\vec{\nabla}_{p} \times \vec{A} = \vec{r}(\varphi_{p}) \left[\frac{1}{r_{p}} \frac{\partial A_{z}}{\partial \varphi_{p}} - \frac{\partial A_{\varphi}}{\partial z_{p}} \right] + \vec{\varphi}(\varphi_{p}) \left[\frac{\partial A_{r}}{\partial z_{p}} - \frac{\partial A_{z}}{\partial r_{p}} \right] + \vec{z} \frac{1}{r_{p}} \left[\frac{\partial (r_{p}A_{\varphi})}{\partial r_{p}} - \frac{\partial A_{r}}{\partial \varphi_{p}} \right]$$
$$\vec{\nabla}_{p} \times \frac{\sin(\varphi_{p} - \varphi_{s}) \hat{r}(\varphi_{p}) + \cos(\varphi_{p} - \varphi_{s}) \hat{\varphi}(\varphi_{p})}{\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1}) \right]^{\frac{1}{2}}} =$$

The cross product was solved using the following relations

$$A_{r} = \frac{\sin(\varphi_{p} - \varphi_{s})}{\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2}\right]^{\frac{1}{2}}}$$

$$A_{\varphi} = \frac{\cos(\varphi_{p} - \varphi_{s})}{\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2}\right]^{\frac{1}{2}}}$$

$$A_{z} = 0$$

$$\vec{V}_{p} \times \vec{A} = \vec{r}(\varphi_{p}) \left[-\frac{\partial A_{\varphi}}{\partial z_{p}}\right] + \vec{\varphi}(\varphi_{p}) \left[\frac{\partial A_{r}}{\partial z_{p}}\right] + \vec{z} \frac{1}{r_{p}} \left[\frac{\partial(r_{p}A_{\varphi})}{\partial r_{p}} - \frac{\partial A_{r}}{\partial \varphi_{p}}\right]$$

Solving for
$$\vec{r}(\varphi_p) \left[-\frac{\partial A_{\varphi}}{\partial z_p} \right]$$
:
 $\vec{r}(\varphi_p) \left[-\frac{\partial A_{\varphi}}{\partial z_p} \right] = -\left[\frac{\partial}{\partial z_p} \left(\frac{\cos(\varphi_p - \varphi_s)}{\left[r_p^2 + a_1^2 - 2a_1r_p\cos(\varphi_p - \varphi_s) + \left(z_p - d_1\right)^2 \right]^{\frac{1}{2}}} \right) \right]$

$$Where: Where: = \frac{\partial A_{\varphi}}{\partial z_{p}} = \frac{\left(\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2}\right]^{\frac{1}{2}}\right)\left(\frac{\partial}{\partial z_{p}}(\cos(\varphi_{p} - \varphi_{s}))\right)}{\left[\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2}\right]^{\frac{1}{2}}\right]^{2}} + \frac{-\cos(\varphi_{p} - \varphi_{s})\frac{\partial}{\partial z_{p}}\left(\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2}\right]^{\frac{1}{2}}\right)}{\left[\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2}\right]^{\frac{1}{2}}\right]^{2}}\right]^{2}}$$

$$\frac{\partial A_{\varphi}}{\partial z_{p}} = \frac{-\cos(\varphi_{p} - \varphi_{s})\left(\frac{1}{2}\right)\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2}\right]^{-\frac{1}{2}}(2)(z_{p} - d_{1})}{\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2}\right]^{\frac{2}{2}}}$$

$$\frac{\partial A_{\varphi}}{\partial z_{p}} = \frac{-\cos(\varphi_{p} - \varphi_{s})(z_{p} - d_{1})}{\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2}\right]^{\frac{3}{2}}}$$

$$-\vec{r}(\varphi_{p})\left[\frac{\partial A_{\varphi}}{\partial z_{p}}\right] = \frac{\cos(\varphi_{p}-\varphi_{s})(z_{p}-d_{1})}{\left[r_{p}^{2}+a_{1}^{2}-2a_{1}r_{p}\cos(\varphi_{p}-\varphi_{s})+(z_{p}-d_{1})^{2}\right]^{\frac{3}{2}}}\vec{r}(\varphi_{p})$$

$$\begin{split} & Solving for \, \overline{\varphi}(\varphi_p) \left[\frac{\partial A_r}{\partial z_p} \right] : \\ & \overline{\varphi}(\varphi_p) \left[\frac{\partial A_r}{\partial z_p} \right] = \\ & \frac{\partial A_r}{\partial z_p} = \frac{\partial}{\partial z_p} \frac{\sin(\varphi_p - \varphi_s)}{\left[r_p{}^2 + a_1{}^2 - 2a_1r_p\cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}}} \\ & \frac{\partial A_r}{\partial z_p} = \frac{\left[r_p{}^2 + a_1{}^2 - 2a_1r_p\cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}} \frac{\partial}{\partial z_p}\sin(\varphi_p - \varphi_s)}{\left[\left[r_p{}^2 + a_1{}^2 - 2a_1r_p\cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}} \right]^2} + \\ & + \frac{-\sin(\varphi_p - \varphi_s) \frac{\partial}{\partial z_p} \left[r_p{}^2 + a_1{}^2 - 2a_1r_p\cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}}}{\left[\left[r_p{}^2 + a_1{}^2 - 2a_1r_p\cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}} \right]^2} \\ & \frac{\partial A_r}{\partial z_p} = \frac{-\sin(\varphi_p - \varphi_s) \frac{\partial}{\partial z_p} \left[r_p{}^2 + a_1{}^2 - 2a_1r_p\cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}}}{\left[\left[r_p{}^2 + a_1{}^2 - 2a_1r_p\cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}} \right]^2} \\ & \frac{\partial A_r}{\partial z_p} = \frac{-\sin(\varphi_p - \varphi_s) \left(\frac{1}{2} \right) \left[r_p{}^2 + a_1{}^2 - 2a_1r_p\cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}}}{\left[r_p{}^2 + a_1{}^2 - 2a_1r_p\cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}}} \end{split}$$

$$\frac{\partial A_r}{\partial z_p} = \frac{-\sin(\varphi_p - \varphi_s)(z_p - d_1)}{\left[r_p^2 + a_1^2 - 2a_1r_p\cos(\varphi_p - \varphi_s) + (z_p - d_1)^2\right]^{\frac{3}{2}}}$$

$$\vec{\varphi}(\varphi_{P}) \left[\frac{\partial A_{r}}{\partial z_{p}} \right] = \frac{-\sin(\varphi_{p} - \varphi_{s}) (z_{p} - d_{1})}{\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{3}{2}}}$$

Solving for
$$\vec{z} \left[\frac{1}{r_p} \frac{\partial(r_p A_{\varphi})}{\partial r_p} \right] - \vec{z} \left[\frac{1}{r_p} \frac{\partial A_r}{\partial \varphi_p} \right]$$

Solving for the first partial derivative:

$$\begin{split} \frac{1}{r_p} \frac{\partial(r_p A_{\varphi})}{\partial r_p} &= \frac{1}{r_p} \frac{\partial}{\partial r_p} \left(\frac{r_p \cos(\varphi_p - \varphi_s)}{\left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}}} \right) \\ \frac{\partial(r_p A_{\varphi})}{\partial r_p} &= \frac{\left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}} \frac{\partial}{\partial r_p} \left(r_p \cos(\varphi_p - \varphi_s) \right)}{\left[\left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}} \right]^2} + \\ &+ \frac{-r_p \cos(\varphi_p - \varphi_s) \frac{\partial}{\partial r_p} \left(\left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}} \right)}{\left[\left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}} \right]^2} \\ &= \frac{\partial(r_p A_{\varphi})}{\partial r_p} = \frac{\cos(\varphi_p - \varphi_s) \left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}}}{\left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}}} + \\ &+ \frac{-r_p \cos(\varphi_p - \varphi_s) \left(\frac{1}{2} \right) \left(2r_p - 2a_1 \cos(\varphi_p - \varphi_s) \right) \left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}}}{\left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}}} \end{split}$$

$$\vec{z} \frac{1}{r_p} \frac{\partial (r_p A_{\varphi})}{\partial r_p} = \vec{z} \frac{\cos(\varphi_p - \varphi_s)}{r_p \left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{1}{2}}} + \frac{-r_p \cos(\varphi_p - \varphi_s) (r_p - a_1 \cos(\varphi_p - \varphi_s))}{r_p \left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{3}{2}}}$$

Solving for the second partial derivative:

$$\begin{split} \frac{1}{r_{p}} \frac{\partial A_{r}}{\partial \varphi_{p}} &= \frac{1}{r_{p}} \frac{\partial}{\partial \varphi_{p}} \left(\frac{\sin(\varphi_{p} - \varphi_{s})}{\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{1}{2}} \right)}{\left[\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{1}{2}} \right)} \frac{\partial}{\partial \varphi_{p}} (\sin(\varphi_{p} - \varphi_{s}))}{\left[\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{1}{2}} \right]^{2}} + \\ &+ \frac{-\sin(\varphi_{p} - \varphi_{s})}{\partial \varphi_{p}} \left(\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{1}{2}} \right)}{\left[\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{1}{2}} \right]^{2}} \\ &\frac{\partial A_{r}}{\partial \varphi_{p}} = \frac{\cos(\varphi_{p} - \varphi_{s}) \left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{1}{2}}}{\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{1}{2}}} + \\ &+ \frac{-\sin(\varphi_{p} - \varphi_{s}) \left(\frac{1}{2} \right) \left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{1}{2}}}{\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{1}{2}}} \\ &+ \frac{-\sin(\varphi_{p} - \varphi_{s}) \left(\frac{1}{2} \right) \left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{1}{2}}}{\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{1}{2}}} \\ &+ \frac{-\sin(\varphi_{p} - \varphi_{s}) \left(\frac{1}{2} \right) \left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{1}{2}}}{\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{1}{2}}} \end{aligned}$$

$$-\vec{z}\frac{1}{r_{p}}\frac{\partial A_{r}}{\partial \varphi_{p}} = \vec{z}\frac{-\cos(\varphi_{p}-\varphi_{s})}{r_{p}\left[r_{p}^{2}+a_{1}^{2}-2a_{1}r_{p}\cos(\varphi_{p}-\varphi_{s})+(z_{p}-d_{1})^{2}\right]^{\frac{1}{2}}} + \frac{a_{1}r_{p}\sin(\varphi_{p}-\varphi_{s})^{2}}{r_{p}\left[r_{p}^{2}+a_{1}^{2}-2a_{1}r_{p}\cos(\varphi_{p}-\varphi_{s})+(z_{p}-d_{1})^{2}\right]^{\frac{3}{2}}}$$

Putting the cross-product parts together:

$$\vec{V}_{p} \times \vec{A} = \vec{r}(\varphi_{p}) \left[-\frac{\partial A_{\varphi}}{\partial z_{p}} \right] + \vec{\varphi}(\varphi_{p}) \left[\frac{\partial A_{r}}{\partial z_{p}} \right] + \vec{z} \frac{1}{r_{p}} \left[\frac{\partial (r_{p}A_{\varphi})}{\partial r_{p}} - \frac{\partial A_{r}}{\partial \varphi_{p}} \right]$$
$$\vec{V}_{p} \times \vec{A} = \frac{\cos(\varphi_{p} - \varphi_{s}) (z_{p} - d_{1})}{\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{3}{2}}} \vec{r}(\varphi_{p}) + \frac{-\sin(\varphi_{p} - \varphi_{s}) (z_{p} - d_{1})}{\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{3}{2}}} \vec{\varphi}(\varphi_{p}) + \frac{\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2}}{\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{3}{2}}} \vec{z} + \frac{\cos(\varphi_{p} - \varphi_{s})}{r_{p} \left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{3}{2}}} \vec{z} + \frac{\cos(\varphi_{p} - \varphi_{s})}{r_{p} \left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{3}{2}}} \vec{z} + \frac{\cos(\varphi_{p} - \varphi_{s})}{r_{p} \left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{3}{2}}} \vec{z} + \frac{\cos(\varphi_{p} - \varphi_{s})}{r_{p} \left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{3}{2}}} \vec{z} + \frac{\cos(\varphi_{p} - \varphi_{s})}{r_{p} \left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{3}{2}}} \vec{z} + \frac{\cos(\varphi_{p} - \varphi_{s})}{r_{p} \left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{3}{2}}} \vec{z} + \frac{\cos(\varphi_{p} - \varphi_{s})}{r_{p} \left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{3}{2}}} \vec{z} + \frac{\cos(\varphi_{p} - \varphi_{s})}{r_{p} \left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{3}{2}}} \vec{z} + \frac{\cos(\varphi_{p} - \varphi_{s})}{r_{p} \left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{3}{2}}} \vec{z} + \frac{\cos(\varphi_{p} - \varphi_{s})}{r_{p} \left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d_{1})^{2} \right]^{\frac{3}{2}}} \vec{z} + \frac{\cos(\varphi_{p} - \varphi_{s})}{r_{p} \left[r_{p}^{2} + 2a_{1}^{2} + 2a_{1}^$$

$$+\frac{-r_{p}\cos(\varphi_{p}-\varphi_{s})(r_{p}-a_{1}\cos(\varphi_{p}-\varphi_{s}))}{r_{p}\left[r_{p}^{2}+a_{1}^{2}-2a_{1}r_{p}\cos(\varphi_{p}-\varphi_{s})+(z_{p}-d_{1})^{2}\right]^{\frac{3}{2}}}\vec{z} + \frac{\cos(\varphi_{p}-\varphi_{s})}{r_{p}\left[r_{p}^{2}+a_{1}^{2}-2a_{1}r_{p}\cos(\varphi_{p}-\varphi_{s})+(z_{p}-d_{1})^{2}\right]^{\frac{1}{2}}}\vec{z} + \frac{-a_{1}r_{p}\sin(\varphi_{p}-\varphi_{s})}{r_{p}\left[r_{p}^{2}+a_{1}^{2}-2a_{1}r_{p}\cos(\varphi_{p}-\varphi_{s})+(z_{p}-d_{1})^{2}\right]^{\frac{3}{2}}}\vec{z}$$

Putting the cross-product result into magnetic flux density equation:

$$\begin{split} \vec{B}(\vec{r}_p) &= \frac{\mu_0 l_1 N_1 a_1}{4\pi} \Biggl[\vec{r}(\varphi_p) \int_{\varphi_s=0}^{2\pi} \frac{\cos(\varphi_p - \varphi_s) (z_p - d_1)}{\left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{3}{2}}} d\varphi_s \\ &+ \vec{\varphi}(\varphi_p) \int_{\varphi_s=0}^{2\pi} \frac{-\sin(\varphi_p - \varphi_s) (z_p - d_1)}{\left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{3}{2}}} d\varphi_s \\ &+ \vec{z} \int_{\varphi_s=0}^{2\pi} \frac{\cos(\varphi_p - \varphi_s)}{r_p \left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{3}{2}}} d\varphi_s \\ &+ \vec{z} \int_{\varphi_s=0}^{2\pi} \frac{-r_p \cos(\varphi_p - \varphi_s) (r_p - a_1 \cos(\varphi_p - \varphi_s))}{r_p \left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{3}{2}}} d\varphi_s \\ &+ \vec{z} \int_{\varphi_s=0}^{2\pi} \frac{-\cos(\varphi_p - \varphi_s)}{r_p \left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{3}{2}}} d\varphi_s \\ &+ \vec{z} \int_{\varphi_s=0}^{2\pi} \frac{a_1 r_p \sin(\varphi_p - \varphi_s)^2}{r_p \left[r_p^2 + a_1^2 - 2a_1 r_p \cos(\varphi_p - \varphi_s) + (z_p - d_1)^2 \right]^{\frac{3}{2}}} d\varphi_s \end{aligned}$$

The equation was simplified to obtain the final form of the magnetic flux density at a point off the *z*-axis for the top coil.

$$\vec{B}(\vec{r}_{p}) = \frac{\mu_{0}I_{1}N_{1}a_{1}}{4\pi} \left[\vec{r}(\varphi_{p}) \int_{\varphi_{s}=0}^{2\pi} \frac{\cos(\varphi_{p}-\varphi_{s})(z_{p}-d_{1})}{\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p}-\varphi_{s}) + (z_{p}-d_{1})^{2} \right]^{\frac{3}{2}}} d\varphi_{s} \right. \\ \left. + \vec{\varphi}(\varphi_{p}) \int_{\varphi_{s}=0}^{2\pi} \frac{-\sin(\varphi_{p}-\varphi_{s})(z_{p}-d_{1})}{\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p}-\varphi_{s}) + (z_{p}-d_{1})^{2} \right]^{\frac{3}{2}}} d\varphi_{s} \right. \\ \left. + \vec{z} \int_{\varphi_{s}=0}^{2\pi} \frac{-r_{p}\cos(\varphi_{p}-\varphi_{s})(r_{p}-a_{1}\cos(\varphi_{p}-\varphi_{s}))}{r_{p}\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p}-\varphi_{s}) + (z_{p}-d_{1})^{2} \right]^{\frac{3}{2}}} d\varphi_{s} \right. \\ \left. + \vec{z} \int_{\varphi_{s}=0}^{2\pi} \frac{a_{1}r_{p}\sin(\varphi_{p}-\varphi_{s})^{2}}{r_{p}\left[r_{p}^{2} + a_{1}^{2} - 2a_{1}r_{p}\cos(\varphi_{p}-\varphi_{s}) + (z_{p}-d_{1})^{2} \right]^{\frac{3}{2}}} d\varphi_{s} \right]$$

It is observed that $N_1 = N_2$, $I_1 = I_2 = I$, $d_1 = d_2 = d$, $a_1 = a_2 = a$

Equation #1:

$$\vec{B}(\vec{r}_{p}) = \frac{\mu_{0}INa}{4\pi} \left[\vec{r}(\varphi_{p}) \int_{\varphi_{s}=0}^{2\pi} \frac{\cos(\varphi_{p} - \varphi_{s})(z_{p} - d)}{\left[r_{p}^{2} + a^{2} - 2ar_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d)^{2}\right]^{\frac{3}{2}}} d\varphi_{s} \right]$$

$$+ \vec{\varphi}(\varphi_{p}) \int_{\varphi_{s}=0}^{2\pi} \frac{-\sin(\varphi_{p} - \varphi_{s})(z_{p} - d)}{\left[r_{p}^{2} + a^{2} - 2ar_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d)^{2}\right]^{\frac{3}{2}}} d\varphi_{s}$$

$$+ \vec{z} \int_{\varphi_{s}=0}^{2\pi} \frac{-r_{p}\cos(\varphi_{p} - \varphi_{s})(r_{p} - a\cos(\varphi_{p} - \varphi_{s}))}{r_{p}\left[r_{p}^{2} + a^{2} - 2ar_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d)^{2}\right]^{\frac{3}{2}}} d\varphi_{s}$$

$$+ \vec{z} \int_{\varphi_{s}=0}^{2\pi} \frac{ar_{p}\sin(\varphi_{p} - \varphi_{s})^{2}}{r_{p}\left[r_{p}^{2} + a^{2} - 2ar_{p}\cos(\varphi_{p} - \varphi_{s}) + (z_{p} - d)^{2}\right]^{\frac{3}{2}}} d\varphi_{s}$$

$$Equation 2:$$

$$48.445 * 10^{-6} * 0.05 = B_{5\%BEarth}$$

 $B_{5\% BEarth} = 2.42 \mu T$

Helmholtz Coil MATLAB Implementation, Code and Graphs

Referenced formulas for the implementation of the MATLAB code obtained from "General Relation for the Vector Magnetic Field of a Circular Current Loop" by Dr. Robert Schill.

Taylor Expansion for Elliptic Integral:

$$\begin{split} K(k) &\approx \frac{\pi}{2} + \frac{\pi}{8} \, k^2 + \frac{9\pi}{128} \, k^4 \\ E(k) &\approx \frac{\pi}{2} - \frac{\pi}{8} \, k^2 - \frac{3\pi}{128} \, k^4 \\ , \, \text{where} \quad k_c^2 &= \frac{4ar_c}{(r_c + a)^2 + (z - z_o)^2}. \end{split}$$

Magnetic field for R and Z components using the above expansion:

$$B_{r_c}(r_c, \varphi, z) = \frac{\mu_0 I_o}{2\pi} \frac{(z - z_o)}{r_c [(r_c + a)^2 + (z - z_o)^2]^{1/2}} \cdot \left[-K(k_c) + \frac{r_c^2 + a^2 + (z - z_o)^2}{(r_c - a)^2 + (z - z_o)^2} E(k_c) \right]$$
(B1)
$$B_z(r_c, \varphi, z) = \frac{\mu_0 I_o}{2\pi [(r_c + a)^2 + (z - z_o)^2]^{1/2}} \cdot \left[K(k_c) - \frac{r_c^2 - a^2 + (z - z_o)^2}{(r_c - a)^2 + (z - z_o)^2} E(k_c) \right].$$
(B2)

%EE330 Helmholtz coil Matlab %Using taylor expansion of elliptic integral for matlab integration %compatibility % create global value for permeability of free space global u0 u0=4*pi*1e-7; % Set parameters for our model N = 88; %Coil turns IO= 0.0035; %Coil current in Amps (We adjust current to .09A for equivalent earth's mag field) a=.12; %Coil radius in meters % Assign the center of each coil their own coordinate positions xp1=0; yp1=0; zp1=0; % coil 1 xp2=0; yp2=0; zp2=a; % coil 2 % Create axis' where we will observe the Magnetic Field % z and y coordinates are broken into 25 values and observed from %(-0.01:0.01) for y and (0.05:0.07) for z x=0; [y,z]=meshgrid(linspace(-0.01,0.01,25),linspace(0.05,0.07,25)); % setting up radial components rc1=((x-xp1).^2+(y-yp1).^2).^.5; %magnitude of radial vector rc2=((x-xp2).^2+(y-yp2).^2).^.5; %magnitude of radial vector % Set parameters for using Elliptic Integrals k1=(4.*a.*rc1).*(((rc1+a).^2)+((z-zp1).^2)).^(-1); %This is a parameter for calculating the Elliptical integrals kElipt1=(pi/2)+(pi/8).*k1+(9*pi/128).*k1.^2; %taylor expansion of the K elliptical integral. eElipt1=(pi/2)+(-pi/8).*k1+(-3*pi/128).*k1.^2;%taylor expansion of the E elliptical integral. k2=(4.*a.*rc2).*(((rc2+a).^2)+((z-zp2).^2)).^(-1); kElipt2=(pi/2)+(pi/8).*k2+(9*pi/128).*k2.^2; eElipt2=(pi/2)+(-pi/8).*k2+(-3*pi/128).*k2.^2; % Calculate the radial component for each loop Br1=(u0.*N.*I0./(2.*pi.*rc1)).*(z-zp1).*((((rc1+a).^2)+((z-zp1).^2))^(-.5)).*(-kElipt1+eElipt1.*((rc1.^2+a.^2+(z-zp1).^2)./(((rc1-a).^2)+((z-zp1).^2)))); % radial component of B Br2=(u0.*N.*I0./(2.*pi.*rc2)).*(z-zp2).*((((rc2+a).^2)+((z-zp2).^2)) . .^(-.5)).*(-kElipt2+eElipt2.*((rc2.^2+a.^2+(z-zp2).^2)./(((rc2-a).^2)+((z-zp2).^2)))); %∠ radial component of B % Calculate the z component for each loop Bz1=(u0.*N.*I0./(2.*pi)).*((((rc1+a).^2)+((z-zp1).^2)).^(-.5)).*(kElipt1-eElipt1.* ... ((rc1.^2-a.^2+(z-zp1).^2)./(((rc1-a).^2)+((z-zp1).^2)))); %Z component of B1 Bz2=(u0.*N.*I0./(2.*pi)).*(((((rc2+a).^2)+((z-zp2).^2)).^(-.5)).*(kElipt2-eElipt2.* ... ((rc2.^2-a.^2+(z-zp2).^2)./(((rc2-a).^2)+((z-zp2).^2)))); %Z component of B2

% Obtain the cartesian components from the radial components of Br1,2

```
Bx1=Br1.*(x-xp1)./rc1;
By1=Br1.*(y-yp1)./rc1;
Bx2=Br2.*(x-xp2)./rc2;
By2=Br2.*(y-yp2)./rc2;
% Add all components together
Bx=Bx1+Bx2;
By=By1+By2;
Bz=Bz1+Bz2;
% eliminate any instances of infinite
Bx(isnan(Bx)) = 0 ;By(isnan(By)) = 0 ;Bz(isnan(Bz)) = 0 ;
% Calculate the magnitude of the vector for graphability
B_mag=sqrt(Bx.^2+By.^2+Bz.^2);
% Plot
figure
surf(y,z,B_mag)
title('Magnetic Field Magnitude Less Than 5%')
xlabel('Zp')
ylabel('Rp')
zlabel('Bz')
```





Graph of magnetic field magnitude when coil intensity is equal to 5% of the magnetic field of Earth's.



Graph of magnetic field magnitude when coil intensity is equal to the magnetic field of Earth's.

Solenoid Matlab Code and Graphs

Solenoid Theory using Ampere's Circuital Law

$$\oint \vec{H} \cdot d\vec{l} = I_{enc}$$
$$d\vec{l} = dx\hat{x}$$
$$\int_{0}^{L} \vec{H} \cdot dx\hat{x} = I_{enc}$$

 \vec{H} is in same direction as dl in order to survive dot product

$$\int_{0}^{L} \vec{H} dx = I_{s} N_{s}$$
$$\vec{H} (L - 0) = I_{s} N_{s}$$
$$\vec{H} L = I_{s} N_{s}$$
$$\vec{H} = \frac{I_{s} N_{s}}{L}$$
$$\vec{B} = \mu_{0} \vec{H}$$

Equation 3:

$$\vec{B} = \frac{\mu_0 I_s N_s}{L}$$

%solenoid B vs I %bs is mag field of solenoid %u is permittivity of core %L is length of solenoid %N is number of turns %I is current %V is voltage of battery %R is resistor connected to solenoid V = 5.6R = 167N = 115I = V/Ru = 4*pi*10^-7 L = 0.1Is = 0:0.001:I B = (u*N*Is)/Lfigure(1) plot(B) title ("B vs I") xlabel ("I (amps)") ylabel ("B (tesla)")



Graph of B vs I for the solenoid

Compass MATLAB Code and Graphs

```
clc; clear all; close all;
Bearth = 48.445 * 10 ^ -6; %Magnetic field of earth at UNLV
Bwire = (48.445 * 10 ^ -6) * 0.05; % 5% of magnetic field in our coil
d_angle = atand(Bwire/Bearth);
fprintf('The magnetic field of the earth at UNLV is (in Teslas): %d\n', Bearth)
fprintf('\n')
fprintf('5 percent of the magnetic field in our wire must be less than (in Teslas): %
d\n', Bwire)
fprintf('\n')
fprintf('The compass must not deviate more than (in degrees): %f\n', d_angle)
%Create vectors for magnetic fields of Earth, the coil, and the compass
p0 = [0 0];
p1 = [0 Bearth];
vectarrow(p0,p1)
hold on
p2 = [0 \ 0];
p3 = [Bwire 0];
vectarrow(p2, p3)
hold on
p4 = [0 0];
p5 = [2.4*10^-6 Bearth];
vectarrow(p4, p5)
hold on
%include vector to set scaling of y-axis
p6 = [0 0];
p7 = [Bearth 0];
vectarrow(p6,p7)
hold on
title('Compass Deviation Due To Helmholtz Coil')
ylabel('Magnetic field of Earth')
xlabel('Magnetic field of Coil')
```

Output of code:

The magnetic field of the earth at UNLV is (in Teslas): 4.844500e-05 5 percent of the magnetic field in our wire must be less than (in Teslas): 2.422250e-06

The compass must not deviate more than (in degrees): 2.862405



Graph of the Compass deviation due to the coil and Earth's magnetic field. We see the compass must not deviate past 2.86°.

Resistance Calculations:

```
clc; clear all
Ro1 = 1.44e-6; %the resistivity of the Kanthal A-1 wire
L1 = 1.7111; %length of wire in meters
A1 = 3.2e-8; %cross sectional area of a 32 guage wire (32nm)
Kanthal_Resistance = (Ro1*L1)/A1 %this is the resistance of the Kanthal resistor
Ro2 = 60e-5; %resistivity of graphite
L2 = 0.095; %length of graphite resistor
A2 = 0.000000038; %a very thin sheet of graphite on paper (approximated based off
%of multimeter measurement)
Graphite_Resistance = (Ro2 * L2) / A2 %resistance of graphite on paper resistor
```

```
Kanthal_Resistance =
```

76.9995

Graphite_Resistance =

1.5000e+03

Note: The cross-sectional area of the graphite resistor is approximate since it is difficult to measure.